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Wavelets in Control Engineering

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(Abstract) This article for control engineering describes new analysis and filter technique: the wavelet transform. The technique presents the signal on both a time as frequency axis, comparable with a windowed Fourier transformation that is shifted along the time axis. In Fourier techniques window width is changed as function of analyzing frequency. Furthermore there is much freedom in choosing the analysis function, which let the wavelet transform do more than only to discover frequency information. Besides as analysis technique the wavelet transform can be used for filtering purposes. With the discrete version of the wavelet transform (DWT) decomposition in frequency-dependent coefficients is possible, from which the original signal can be reconstructed. Since the origins of the DWT lie in the field of signal processing, the available algorithms are efficient but not optimal with respect to the delay times. Therefore a real-time wavelet filter algorithm is derived, which can also be used for analyzing purposes. Two application are worked out ; 1. Wavelet filters for encoder quantization denoising and 2. Online feature detection on a CD-player setup. Wavelets show good results in isolating features (time-patterns in signals), especially short-living events. Building dedicated waveforms can improve the results and the real-time algorithm makes fast detection possible. The experiences are validated on a servo-loop of a CD-player setup, on which external disturbances are presented in the form of shocks and disc-faults. For some disturbances very early detection is possible: the wavelet filter already isolates features where the measurement still moves within the noise level.

Keywords: Fourier Transformations; Frequency Analysis; Wavelet Filter; DWT.

1. Introduction

From an historical point of view, wavelet analysis is a new method, though its mathematical backgrounds date back to the work of Joseph Fourier in the nineteenth century. Fourier laid the foundations of frequency analysis, which proved to be enormously important and influential. The first recorded mention of what we now call a *wavelet* seems to be in 1909, in a thesis by Alfred Haar. The concept of wavelets in its present theoretical form was first proposed by Jean Morlet. The methods of wavelet analysis have been developed mainly by Y. Meyer and his colleagues, who have ensured the methods dissemination. The main algorithm dates back to the work of Stephane Mallat in 1988. Since then, research on wavelets has become international. With the work of Ingrid Daubechies the mathematical backgrounds of wavelets are as good as covered. It is now time to explore the use of wavelets and find new applications. With this report a chance is taken to find new applications in control engineering. This article focuses on a different use of wavelets in control. Wavelets are studied as a signal processing technique, with good analysis and filter properties. To understand wavelets, some basic knowledge of another technique is needed, i.e. filter banks. Because all these techniques are born in the field of signal processing, all information in literature is presented in a certain style. In the article efforts have been made to use the notations and analogies from the field of control engineering.

2. Time-frequency representations Frequency analysis Fourier transformations

Signal analysis includes extracting of relevant information from a signal by transforming it. Most harmonic analysis tools in control engineering decompose the original signal into orthogonal trigonometric basis functions. Such tools are called Fourier methods because of the ideas of Joseph Fourier (1768-1830) and his determination to get them accepted. However, the origins of these methods predate Fourier. The best known but not the oldest transformation is the **Fourier transform** (FT) as presented in (2.1). This orthogonal decomposition and the inverse transformation (2.2) are defined for continuous signals of infinite length

$$X_{FT}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad \text{-----2.1}$$

The analysis function XFT(f) gives information about the global frequency distribution in a signal (spectral density).

$$x(t) = \int_{-\infty}^{\infty} X_{FT}(f)e^{j2\pi ft}df \quad \text{.....2.2}$$

Many signals, especially periodic, ones do not fulfill first condition so the Fourier transform cannot be applied. Hence for pure *periodic* signals (*x(t)*=*x*(*t*+*T*) the **Fourier series**

(FS)([RM87**]** p.99). can be used

$$X_{FS}(f_n) = \frac{1}{T}\int_0^T x(t)e^{-j2\pi f_n t}dt; \quad f_n = 0, \pm\frac{1}{T}, \pm\frac{2}{T}, \dots \qquad 2.3$$

This transformation shows that pure periodic signals have a discrete frequency spectrum (*f=n/t*). This is trivial because only multiple of aground frequency *I/T* fit exactly in a time period *T*. Reconstruction is also possible

$$x(t) = \sum_{f_n=-\infty}^{\infty} X_{FS}(f_n)e^{j2\pi f_n t}; \quad f_n = 0, \pm\frac{1}{T}, \pm\frac{2}{T}, \dots \qquad 2.4$$

If a signal is *discrete in time* (sampled with sample tim ΔT) and has finite energy the **discrete-time Fourier transforms** (DTFT) and inverse can be calculated

$$X_{DTFT}(f) = \sum_{k=-\infty}^{\infty} x(k)e^{-j2\pi f k\Delta T} \qquad \text{------- } 2.5$$

Due to the sampling of the signal, the frequency spectrum becomes periodic. Therefore the frequency-band can be decreased to the interval between minus and plus the

Half sample frequency: $\left[-\frac{1}{2\Delta T}, \frac{1}{2\Delta T}\right]$. …2.6

$$x(k) = \int_{-\frac{1}{2\Delta T}}^{\frac{1}{2\Delta T}} X_{DTFT}(f)e^{j2\pi f k\Delta T}df \qquad \text{-----} 2.7$$

From this transform we see that an infinitely long sampled signal possesses a continuous frequency spectrum which is periodic. As in the continuous-time case for periodic signals, that have infinite energy, there exists an appropriate transformation known as the **discrete Fourier transform** (DFT) for periodic discrete-time signals:

$$X_{DFT}(f_n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)e^{-j2\pi f_n k\Delta T}; \quad f_n = 0, \frac{1}{T}, \frac{2}{T}, \dots, \frac{N-1}{T} \qquad \dots..2.8$$

This transform holds for discrete-time signal *x(*k) sample at △T and (smallest) period time T. Knowing this we can define N=T/△ T as the the number of samples. Again we see that the spectrum is also periodic, so the frequencies that can be analyzed are finite. The DFT is the only Fourier transform which can be finitely parameterized. The inverse formulation is given by

$$x(k) = \frac{1}{\Delta T}\sum_{f_n=0}^{\frac{N-1}{T}} X_{DFT}(f_n)e^{j2\pi f_n k\Delta T} \qquad \dots.2.9$$

Note that all Fourier transformations present the signal's frequency content as a complex value. In general only the *frequency spectrum* is used, which can be calculated by taking the magnitude of the complex value. Phase information is only interesting when examining periodic signals and can be obtained taking the angle of the transform.

3.Time-frequency analysis Short-time Fourier analysis:

Fourier transformations offer a one-dimensional projection of a frequency spectrum and will only define the notion of global frequency in a signal. The analysis works well on stationary and pseudo-stationary signals. In case of abrupt changes in time or bursts of equal frequency with delays in between, we cannot talk about *the* phase or *the* amplitude of the signal's spectral components. Making a Fourier transformation of such signal, these components will be spread out a little over the whole frequency axis. (Fig.3. a and b). However, one is often more interested in the momentary or local distribution of the energy as a function of frequency.

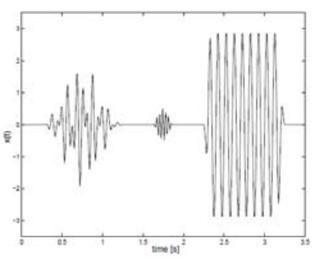


Fig3.(a) Time frequency

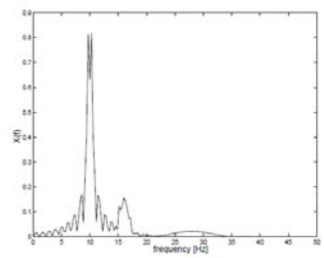


Fig.3. (b) Short time analysis

Fig 3 (a) Signal builds up on 3 harmonics: burst of 10 and 16H: short burst of 28H: (low amplitude) and a burst of 10 H: out of phase with the first10H: part

Fig3 (b) Discrete Fourier transform of signal: 10H: is not exactly detected due to difference in phase (actually the spectrum drops down at 10H :} .16H: is spread out and H: disappears in the noise caused by the transform

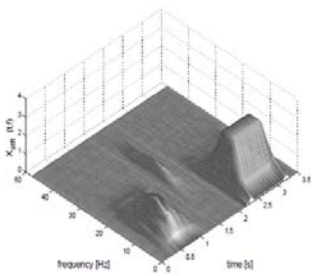
To get time-dependent frequency information of a signal, a two-dimensional representation of the signal is needed, composed of spectral characteristics depending on time. In many reports and books this representation is compared with a musical score: a way to show which tones (frequencies) at what time have to be played. An adapted Fourier transformation for this purpose is known as the **short-time Fourier transforms** (STFT) which also uses windowing 2:

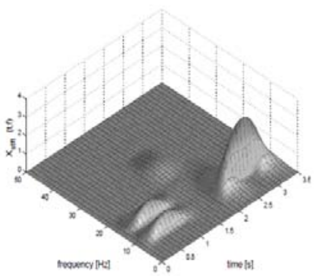
$$X_{STFT}(\tau, f) = \int_{-\infty}^{\infty} x(t)g^*(t-\tau)e^{-j2\pi f t}dt \qquad \text{--}3.1$$

The parameter f in (3.1) is similar to the Fourier frequency and □ represents the central time on which the local Fourier transform is made. Note that (3.1) is defined for continuous time signal with infinite duration, like (2.1). In practice (3.1) is not workable but can be rewritten to another Fourier transform

presented in the previous section. In that way both time and frequency can be discretized. However, the success of the analysis on a certain signal depends strongly on the choice of the window . We can feel that a short window will be able to act very locally in time (good time resolution) but won't be able to discriminate very good between different frequencies. A longer window is less concentrated in time, but can better discriminate between frequencies (see Fig. c and d). The whole problem is to balance the frequency resolution against time resolution.

Conclusion: in signal analysis, we cannot have both good frequency ((Δf)) and good time ((Δt)) resolution. Intuitively we can feel that it is not possible to determine *the* frequencies that exist on a *certain* moment ($(\Delta t \longrightarrow 0)$) of the signal, and vice versa. This is referred to as the uncertainty principle or Heisenberg inequality (see appendix A.1), which states that the bandwidth-time product is lower bounded





(a)

Fig.2(a) Fig.2(b)

Fig.2 (a) Good time resolution, bad frequency resolution

Fig.2 (b) Good frequency resolution. bad time resolution Frequency separation is good, but time location not.

Wavelet analysis

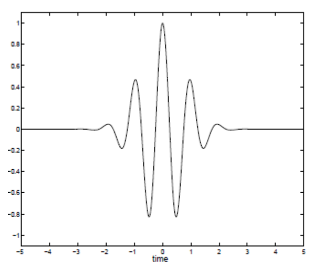
In the case of the STFT a trade-off between time and frequency resolution has to be made. Once a window has been chosen for the STFT, the time-frequency resolution is fixed over the entire time-frequency plane since the same window is used at all frequencies. A fixed frequency/time resolution feels not very natural looking at real world signals. To examine (low-frequent) transients or gross features in a signal a long window is preferable. Similarly, detecting (high-frequent) details or small features will only work well if a small window is used. This property can be achieved by changing the window width with the analyzing frequency. The price we pay using this *multiresolution* approach is a less high frequency resolution, and less time resolution in the lower frequency range. However, looking at the nature of most signals this approach will give better results than a fixed-resolution one. The **wavelet transform** (WT) is developed as an alternative approach to the short-time Fourier transform to make a multi resolution analysis possible

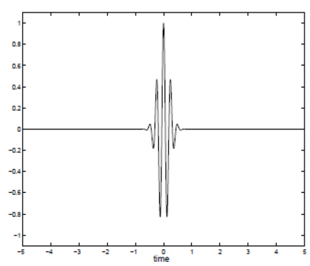
$$X_{WT}(\tau, s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \qquad ..3.2$$

The wavelet analysis is done in a similar way as the STFT analysis, in the sense that the coefficients are determined by measuring similarity between the signal and an analyzing function. The transform is also computed separately for different segments of the time-domain signal, resulting in a

two-dimensional representation. Looking at (3.2) we see an analyzing function $\psi(\frac{t-\tau}{s})$. The *analyzing* shape and *window* shape are defined in one function. Since this function must be oscillatory in some way to be able to discriminate between different frequencies, it is called *wavelet*, which means small wave. The wavelet basis function (or *mother wavelet*) can be chosen in several ways, without the need of using sine-forms as in the Fourier analysis. This freedom makes wavelet analysis not just a harmonic analysis method, as we will see later. The mother wavelet $\psi(t)$ is contracted and dilated by varying3 s, which means changing the

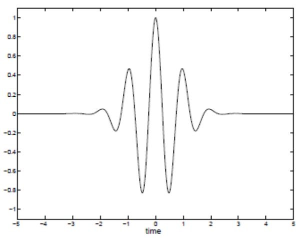
Fig.3 (c) Large scale **Fig.3 (d)** Small scale





scale of our analyzing function $\psi(\frac{t-\tau}{s})$. By varying not only the central analysis frequency but also the effective window width is changed (Fig.3c and d). Therefore, in this multiresolution analysis scale is used instead of frequency: large scale means taking a global view of the signal, so analyzing lower frequencies. Small scale takes a short detailed look, revealing high frequency information. Every scale corresponds to a central analyzing frequency and is inversely proportional to that frequency. In order to normalize the signals energy for every scale, the wavelet coefficients in (3.2) are divided by $\sqrt{|s|}$. The analyzing function $\psi(t)$ is both localized in frequency and time. Therefore, as with the STFT, it can be proven that the bandwidth-time product $\Delta f \Delta t$ is constant (A.4) and lower bounded by the

Heisenberg inequality (A.1). The STFT uses a fixed window width (Fig.3 e and f), so both Δt and Δf are constant. Comparing with the wavelet, we see that the wavelet uses a varying window width $(\psi(t) \rightarrow \psi(\frac{t-\tau}{s}))$ leading to a fixed number of cycles in the analyzing function Increasing s (enlarging the window width) will decrease time resolution Δt inherently resulting in a increase of frequency resolution Δf. Because $\Delta f \Delta t$ is constant



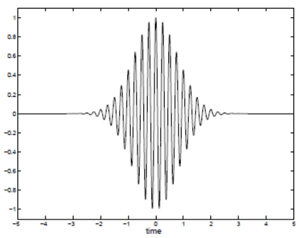
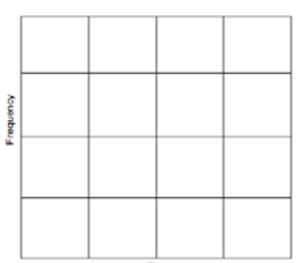


Fig.3 (e) Low frequency **Fig.3 (f)** High frequency

This imposes that Δf is proportional to *f*, or

$$\frac{\Delta f}{f} = c \qquad \text{-------- 3.3}$$

where □ is a constant. So the relative frequency resolution is constant in a wavelet analysis. This property is no renewal: many systems have relative frequency resolution. The perception of our ears has this property and tonal musical is arranged around a logarithmic scale. Transfer functions are presented on a logarithmic grid and normal (LTI) filters have passbands that are defined in dB/octave. Since octaves place the frequency axis in a dyadic grid these filters also have relative frequency resolution. Using (3.3) is a very natural way of presenting frequency-dependent phenomena. This is also known as the *constant Q* property, Q being the quality factor of the filter, defined as center-frequency divided by the bandwidth.



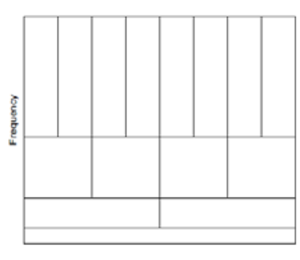


Fig.3 (g) **Fig3 (h)**

Fig.3 (g) Fixed time –frequency resolution

Fig.3 (h) Multi resolution time-frequency plane

For both analysis and filter technique general applications are presented. It is examined if wavelets can be interesting in control engineering. Since the origins of the DWT lie in the field of signal processing, the available algorithms are efficient but not optimal with respect to the delay times. Therefore a real-time wavelet filter algorithm is derived, which can also be used for analyzing purposes. Expressions for the delay time show that such a filter cannot be used as an online controller.

Different time-frequency resolutions. Note that in both cases the product is the same (this can be seen as the area of the blocks above) A possible wavelet basis function could be that of the STFT (3.4). This often used wavelet for analysis purposes is called the *Morlet* wavelet .It is obtained by using a Gaussian (bell shaped)
Window:

$$\psi(t) = g(t)e^{-j2\pi f_c t}, \qquad g(t) = \sqrt{\pi f_b}\, e^{\frac{t^2}{f_b}}$$

------- 3.4

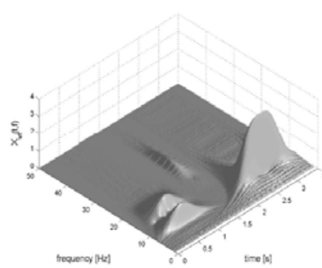
The wavelet center frequency f_c and the band width parameter f_b are tuning parameters. Together they determine the number of cycles in analyzing function. Looking at (3.5) we see that t is replaced by t-T/8, For the Morlet wavelet, scale and frequency are then coupled as:

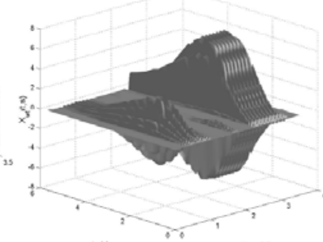
$$f = \frac{f_c}{s} \qquad \text{-------3.5}$$

Comparing figures it is clear that wavelet analysis gives a better view of the frequency content of the signal than one of the separate STFT analysis. The wavelet transform has a good time and poor frequency resolution at high frequencies (small scales), and good frequency and poor time resolution at low frequencies (high scales). In most non-stationary signals high-frequent events take place in a short time span while low-frequencies build up the signal and exist for a longer time.

Fig.3 (i) **Fig.3 (j)**

(a)





Wavelet transform using Market wavelet(signal as in Fig.3.(i). For any comparison with Fig.3(j) easier the analysis is picture along a frequency axis instead of a scaline axisWavelet transform using the Mexican hat

Wavelet analysis can be used to detect other properties than only frequency content of the signal (i.e. [Lew95]). The *Mexican hat* wavelet is a real waveform and isolates local minima and maxima at the different scales . All waveforms have their specific properties and the characteristics that can be

revealed from the signal are always coupled to a certain scale, so they are more or less frequency related. More waveforms for analysis purposes are presented in appendix B.1. Just as the STFT the wavelet also has an inverse transformation; it can be proven that reconstruction of the original signal x(t) is possible using:

$$x(t) = \frac{1}{c_{\psi}^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X_{WT}(\tau, s) \frac{1}{s^{2}} \psi(\frac{t-\tau}{s}) d\tau ds$$

------3.6

Where c_v is the admissibility constant depending on the (mother) wavelet. It must satisfy the following admissibility condition.

$$c_{\psi} = \sqrt{2\pi \int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^{2}}{|\omega|} d\omega} < \infty \qquad \text{--------3.7}$$

in which ψ (w) is the Fourier transform of the analyzing wavelet. This condition implies ψ (0)=0. Which means that the wavelets must have zero mean: $\int_{-\infty}^{\infty} \psi(t) dt = 0.$ Since all wavelet – function candi- dates can be accompanied by a constant to let the integral be zero this is not a very restrictive requirement. Furthermore, to satisfy the wavelet must be at least oscillatory.

4. Applications in control engineering

Having explored the general application fields for wavelets it is now time to look if problems in control can be tackled with either wavelet analysis or the wavelet filter technique. First the advanced filter characteristics of the DWT are presented and denoising is explained in an example. Then the possibilities for in-the-loop applications are investigated, using the results previously. After that, two worked-out applications are presented. The first application uses the filter capacity of the DWT for the denoising of a special class of signals. In the second the analyzing properties of the wavelet transform are used in a real-time feature detection application

5. Wavelet filters properties:

It is clear now that the DWT process can be seen as filtering. Most (discrete) filters normally used in control engineering are linear time-invariant filters. The magnitude of the transfer function changes with frequency and depends linear on the amplitude:in discrete time.

$$|H(s = 2\pi f)| \text{ or } |H(z = e^{j2\pi f})|$$

-----5.1

Normally they have nonlinear phase behaviour. They can have linear phase but then some group delay will be present. With a wavelet filter (and also a filter bank) it is possible to define an arbitrary decay for the amplitude in a certain frequency band. So it is possible to create almost any shape in the magnitude-plot of the transfer function of the filter. In the DWT this is a function of the frequency band (decomposition level p) and the values of the coefficients, representing the amplitudes of the inputs

$$|H(f)| = |\sum_{p} H_{p}(c_{p})| \qquad \text{-----5.2}$$

Nonlinear transfer functions are possible, however very exotic designs are not found in literature, which is probably due to amplitude and phase disturbances at the borders of the frequency bands. The alias and disturbance cancellation for QMF structures is only guaranteed if the coefficients are not processed. Every filter action will create disturbances at the borders. Contrary to most LTI-designs, wavelet filters have linear phase behaviour which results in a certain group delay

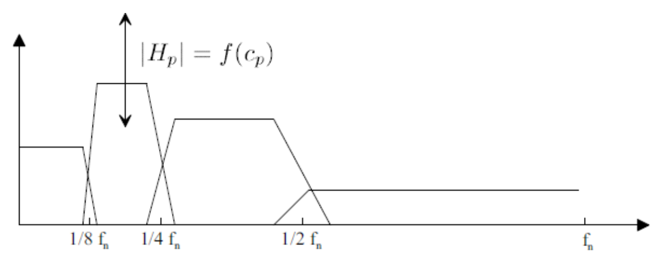


Fig.5 Possible magnitude responses of the filter in the DWT. theoretically, exotic nonlinear filters are possible, since the gain at all levels can be any function of the coefficients.

For off-line filtering or in supervisory loops wavelets can be interesting. Two application are worked out; 1. Wavelet filters for encoder quantization denoising Encoders, widely used in motion systems, always generate noise which is especially annoying when derivatives of the measurement have to be calculated. Often low pass filtering is applied, but if the amplitudes and frequencies of a signal are spread over a broad range, this approach fails

6. DWT example: de noising

In general de noising is accomplished using thresholding. On every level of the e composition a threshold δ is applied. Coefficients below the threshold are omitted in the reconstruction. By doing this, the low amplitude regions in certain frequency bands, presumably due to noise, are suppressed. The noise power is assumed to be smaller than the signal power but also then, it is impossible to filter out all the noise without affecting the signal (Fig.6a and b)

Fig.6(a) Hard thresh holding **Fig.6 (b)** Soft thresh holding

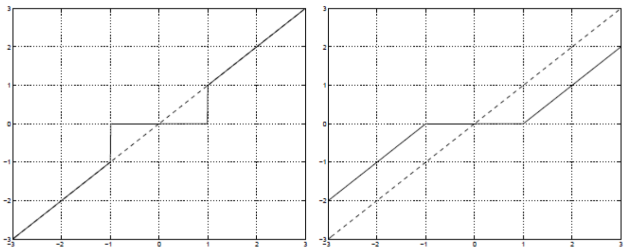


*Fig. (6c). **Wavier filter properties***

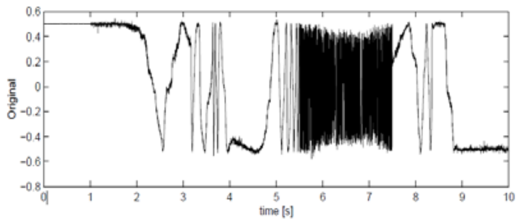


Fig.6(.c): Real –life signal, sampled at 10 kHz.

There are plenty of DWT-waveforms that can be used for denoising. The waveform determines the quality of decomposition since the wavelet coefficients are a measure for the resemblance between signal and wavelet. The best results are obtained when the waveform fits good to the signal. For most signals, especially in control, a smooth waveform must be chosen. Most measurements in control are produced by causal systems which have lowpass behaviour, so their outputs will be smooth. However, there will always be a trade-off between smoothness of the waveforms and computation time, since higher order wavelets are smoother. In this example, more or less arbitrary, the db4 wavelet is chosen. The 8-level decomposition is shown on the next page. From level C1, which projects the highest frequencies, it can be seen that there is not only noise in this frequency band. So it would be impossible to denoise this signal with a low pass filter (Fig.6d).

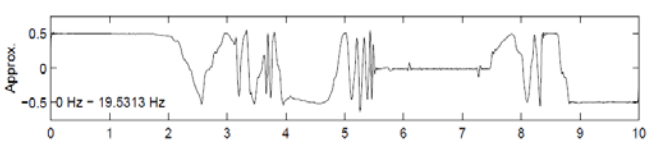


Fig.6d: Approximation of the 6-level decomposition

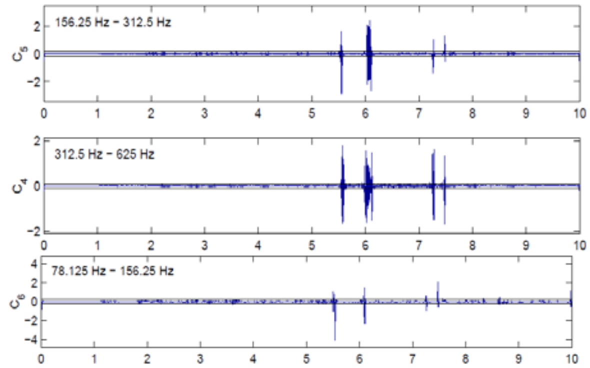
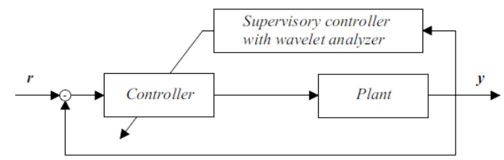


Fig.6(e): Lowest channel signal

From the Approximation it is clear that the basic shape of the signal is composed of frequencies below 20 Hz (Fig. 6e). This signal is obtained by reconstructing only the lowest channel, so all levels in Fig.6d are completely suppressed. This could also be the result of a low pass filter.

7. Wavelets in control loops



(a) Wavelet analysis in a supervisory loop.

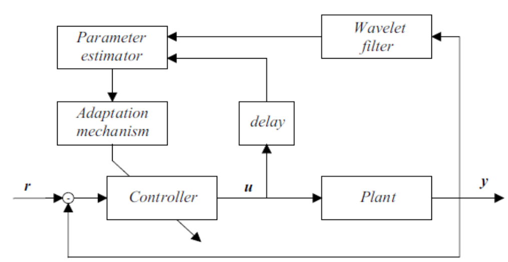


Fig. 7 (a)

Fig. 7(a): Wavelet filter used in an adaptive control setup (self tuning regulator).

The controller output must be delayed by a factor id thegroup delay of the wavelet filter

Sometimes it is desirable to adapt the controller if certain unwanted events disturb the closed-loop. If these events are the result of external disturbances, often the only way to detect them is using a measurement which is part of the closed-loop (Fig 7.(a) and 7(b)).The following is a sample of 2-level wavelet transform applied on a 512 x 512 gray scale bitmap image. the original image (Fig.7c and d)

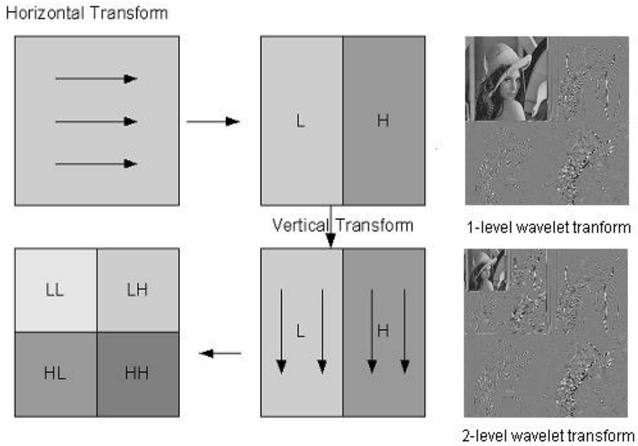


Fig.7(b). Results from applying 2 levels of wavelet on a 512 x 512 bitmap

The idea behind wavelet transform is expressed in Fig.6. Most of the image information is in the low frequency filters. High frequency filters only represent the fine details. For the lossy compression, the idea is to ignore the high level transforms and regenerate your signal using your low frequency filters.

Following is a sample of our retrieved image after decompression

Fig.7(c) Decomposed image

The decompressed image above is identical to the original bitmap image for the small and large sizes. The only exception was the 128 x 128 where we had some distortion in the retrieved image. The size of the matrix for this image is 128 x 128 which makes it hard to find the exact cause for those few pixels. Even for a larger image size like 256 x 256, as it has been shown above are regeneration is close to

Conclusions

Wavelet analysis is a powerful tool for frequency analysis and feature detection. Although feature detection is not really a control issue, it is related to certain control problems. With the wavelet analyzer. it is possible to detect disturbances in real time with maximum time-resolution. It is possible to adapt controllers in a very early stage. The feature detector can be seen as an identification technique for disturbances. For periodical disturbances the optimal waveform in the detector can be adapted, and an adapted feature detector is born. A possible next step is to implement the analyzer in a complete supervisory system on an existing control system, i.e. a CD-player setup. An adaptation algorithm for the controller can be developed, just as the DFT for Fourier analysis, the DWT can be seen as the most efficient, and therefore widely used version of the wavelet transform. Moreover, the reconstruction capability gives the DWT very nice properties if it is used as a filter. However, the delay time is considerable and a wavelet filter cannot be used for control purposes. The optimized denoising technique for encoder signals shows good results. Also for big quantization-levels a smooth output is obtained. Besides thresholding, other possible methods for processing the coefficients can be considered. Therefore, further investigation in the possibilities of nonlinear filtering is desirable. The field of signal processing owns a lot of interesting techniques, also for control engineers. Filter banks are only shortly described; their variety is big and they offer great possibilities for filter purposes. Just as with wavelets, real-time filter banks are possible. In this article only perfect reconstruction orthonormal filter banks and wavelets are mentioned. So more research to dedicated filter banks is recommended. To construct filter banks, the theory must be studied closely which will take a lot of time.

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